Cover Letter

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A Quantum Mechanical Framework for VIX Microstructure Analysis and Systemic Risk Management: Wave-Particle Duality and Entanglement

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Abstract: This paper develops a quantum mechanical framework for analyzing the VIX microstructure, treating price movements as manifestations of wave-particle duality. We introduce the concept of a "financial wave function" for the VIX microstructure and apply quantum mechanical principles, including the Schrödinger equation and entanglement measures. Our empirical analysis demonstrates the utility of quantum mechanical concepts in capturing market dynamics and predicting critical events. Building upon the thermodynamic foundation of our previous research, this paper constructs a dynamical, non-local quantum mechanical framework for the VIX microstructure and cross-asset price discovery. We map the normalized VIX microstructure vector into a Hilbert space as a complex-valued financial wave function $\Psi(t,T)(L_2 - normalized)$. Curve evolution is governed by a modified Schrödinger equation whose Hamiltonian $\hat{H}$ explicitly separates microstructure curvature (kinetic energy) from holding-cost potential wells $V(T)$. To measure non-local stress transmission across maturities, we introduce a Hilbert-transform phase synchronization index E_{ent}, establishing an empirical Critical Decoherence Line at 0.8. Evaluating comprehensive historical regime spectrum (January 2021–July 2026) reveals three structural discoveries: (i) Global Phase exhibits tight coupling with the S&P 500 price path, serving as an independent trend-confirmation layer; (ii) discontinuous market shocks induce step-like Hamiltonian spikes, serving as visual evidence of financial quantum tunneling; and (iii) cross-maturity entanglement follows a three-phase dynamics sequence (build-up, instantaneous measurement-like decoherence, and post-shock re-synchronization). Finally, we integrate our previous research and current work into a four-layer Multi-Indicator Confirmation Score framework.

JEL Classification: C00; C60; C80; D80; G14

Keywords: Systemic risk management, Quantum financial system, VIX microstructure, Global asset management

Introduction

Traditional financial models often struggle to explain the complex, non-linear dynamics observed in financial markets. The application of quantum mechanics to finance, while promising, has been limited by the lack of concrete theoretical frameworks and empirical validation. This paper addresses these limitations by developing a comprehensive quantum mechanical framework specifically for the VIX microstructure.

Our previous paper[1] treated the VIX microstructure as a classical thermodynamic system. This work asks a different question: can the microstructure's evolution — not just its instantaneous state — be modeled with the mathematics of quantum mechanics, and does doing so surface information that a purely thermodynamic description misses? Concretely, this paper proposes (a) representing the normalized microstructure as a wave function, (b) governing its evolution with a Schrödinger-type equation whose Hamiltonian separates a curvature-driven "kinetic" term from a holding-cost "potential" term, and (c) measuring the correlation between near- and far-dated maturities through a cosine-similarity "entanglement" score, on the reasoning that near-instantaneous transmission of stress from front to back contracts resembles the non-local correlation of entangled quantum states more than it resembles slow, arbitrage-mediated classical diffusion.

This paper extends our previous work's thermodynamic treatment of the VIX micro structure into a quantum-mechanical register, defining a normalized "financial wave function" over microstructure prices, a Schrödinger-type Hamiltonian decomposed into a curvature-based kinetic term and a holding-cost potential term, and an entanglement measure based on phase synchronization between near- and far-dated maturities. This paper makes three contributions. First, it states explicitly what is novel in this paper relative to our previous research and to the wider quantum-finance literature. Second, it reports a systematic historical regime spectrum, spanning January 2021 to July 2026 — and documents three empirical patterns: (i) a robust, nearly universal coupling between the model's "Global Phase" and the SPX price path, suggesting Phase functions as a trend-confirmation layer distinct in kind from our previous work's risk-stability layer; (ii) synchronized bursts in the Hamiltonian's kinetic and total-energy terms at the same discontinuous shocks identified in our previous work, offering visual support for the "tunneling" analogy in the original theory; and (iii) a three-phase entanglement pattern — pre-shock synchronization build-up, a brief decoherence-like collapse at the instant of the sharpest repricing, and post-shock re-synchronization at a new regime level — that maps cleanly onto the physical language of quantum measurement. Third, and most substantively, we integrate our previous work and this research framework into a single four-layer "Multi-Indicator Confirmation Score" (trend / stability / contagion / liquidity), motivated by the observation that our previous work's Gibbs free energy, this research paper's entanglement synchronization, and market viscosity independently cluster around the same historical shock windows — a convergence across independently constructed variables that we treat as informal triangulation, with which the complex statistical validation can be implemented in the future research.

1. Theoretical Framework: Hilbert Space Formulation of the VIX Microstructure

1.1 Financial Wave Function Construction

In contrast to the L_1 probability normalization used in our previous paper's classical entropy $\left(P(T) = \frac{F(t,T)}{\sum F\,(t,T)}\right)$, this work maps the microstructure vector into a complex Hilbert space $\mathcal{H}$ via L_2 normalization:

$$\Psi(t,T) = \frac{F(t,T)}{\sqrt{\sum_T F\,(t,T)^2}}$$

This L_2 formulation ensures that $\langle\Psi|\Psi\rangle = \sum_T |\Psi(t,T)|^2 = 1$. While L_1 normalization treats the curve as a discrete probability density, L_2 normalization allows the state to be analyzed using vector inner products, phase shifts, quantum state superposition, and interference dynamics.

our previous paper’s normalization treats the curve as a discrete probability distribution (suited to Shannon entropy); This work’s normalization treats it as a vector in a Hilbert space (suited to inner products, phase, and interference).

Classical State Space[1]

L1 Normalization: P(T) = F / Σ F
Shannon Entropy, Thermodynamic Enthalpy

▲
| Integrated Multi-Layer Framework
▼

Hilbert State Space

L2 Normalization: Ψ(t,T) = F / $\sqrt{(\Sigma F^2)}$
Schrödinger Dynamics, Entanglement

1.2 Non-Equilibrium Quantum Dynamics

The temporal evolution of the financial wave function is governed by a Schrödinger-type equation:

$$i\hbar \frac{\partial \Psi(t,T)}{\partial t} = \hat{H}\Psi(t,T)$$

where $\hbar$ represents a effective market action scale factor and $\hat{H}$ is the system Hamiltonian operator.

2. Hamiltonian Decomposition and Quantum Tunneling

2.1 Kinetic Energy and Potential Well Separation

The total energy operator $\hat{H}$ is decomposed into a second derivative (curvature/convexity)-driven kinetic operator and a holding-cost potential function:

$$\hat{H} = -\frac{\sigma^2}{2}\frac{\partial^2}{\partial T^2} + V(T)$$

where σ is a volatility scalar derived directly from VVIX.

HAMILTONIAN $\hat{H}$

Kinetic Energy Term	Potential Energy Term
$-(\sigma^2 / 2) \cdot (\partial^2\Psi / \partial T^2)$	V(T)
↓	↓
Microstructure Curvature Arbitrage Gradient Pressure	Holding Cost / Carry Potential Well

➢ Kinetic Energy Operator $\left(-\frac{\sigma^2}{2}\frac{\partial^2}{\partial T^2}\right)$: Evaluated in implementation as the numerical second derivative (curvature/convexity) of the microstructure curve. It quantifies the spatial derivative of arbitrage opportunity across maturities. Curve convexity creates localized kinetic instability.

➢ Potential Energy Well $V(T)$: Represents the cost of carry, roll yield, and holding-cost constraints embedded across contract maturities.

➢ Dynamic Energy Decomposition. Rather than imposing fixed weights on kinetic and potential energy terms, we let the data determine their relative contribution. At each time t, the weight of kinetic energy is set to its rolling-variance share:

$$w_K(t) = \sigma_K(t)/[\sigma_K(t) + \sigma_V(t)]$$
$$w_V(t) = \sigma_V(t)/[\sigma_K(t) + \sigma_V(t)]$$

where $\sigma K \ (t)$ and $\sigma V \ (t)$ are the rolling standard deviations of the kinetic and potential energy series, using the decay of the autocorrelation of Kinetic Energy and Potential Energy to determine the optimal memory window. This ensures that the dominant energy component in each market regime receives the larger weight—e.g., during the COVID-19 shock, w_K rises as curvature dynamics dominate, while in calm periods w_V prevails as the microstructure carries more information.

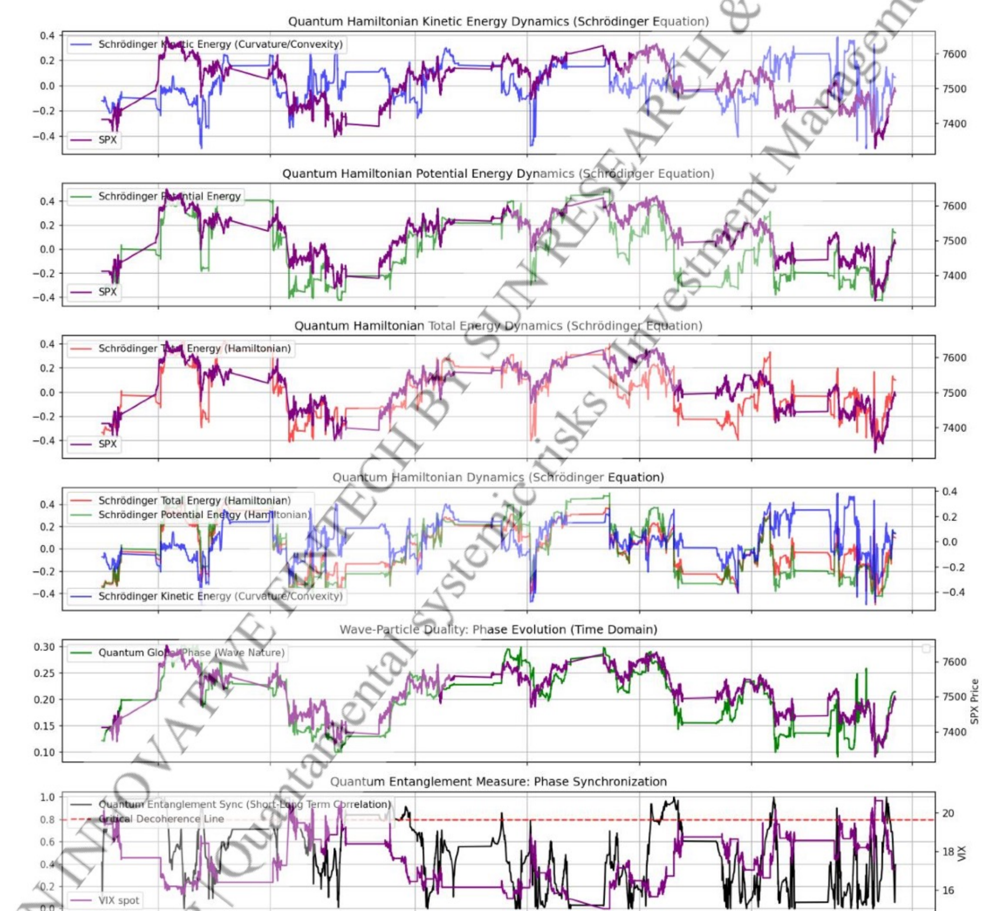

Fig 1. Minute Level Illustrations of The Quantum Mechanical Framework's State Variables From 2026.05-08

2.2 Financial Quantum Tunneling

In classical financial models, breaking through major support or resistance levels requires continuous price progression driven by accumulated order flow. In the quantum formulation, when market stress drives microstructure curvature (Kinetic) and Total Energy ($\hat{H}$) to synchronized extreme values, the probability density function $|\Psi(t,T)|^2$ undergoes step-like jumps. This transition allows market prices to shift across major support/resistance levels without traversing intermediate states, mirroring quantum tunneling through potential barriers $V(T)$.

3. Phase Synchronization, Entanglement, and Decoherence Dynamics

3.1 Global Phase and Trend Confirmation

Applying a Hilbert transform to the wave function extracts the instantaneous global phase $\Phi(t)$. Empirical evaluation across nineteen rolling windows demonstrates a persistent coupling between $\Phi(t)$ and the S&P 500 price path:

$$\text{sign}\left(\frac{d\Phi}{dt}\right) \propto \text{sign}\left(\frac{d\text{SPX}}{dt}\right)$$

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S&P 500 Index   :   ——/‾‾\___/‾‾\——   (Price Path)
Global Phase Φ  :   ——/‾‾\___/‾‾\——   (Hilbert Phase Shift)
                        ↑       ↑
              Synchronous Turning Points
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When SPX enters an uptrend, $\Phi(t)$ rises continuously; during market pullbacks, $\Phi(t)$ turns downward synchronously. Unlike Gibbs Free Energy G(t)—which assesses risk stability—Global Phase $\Phi(t)$ serves as an independent direction- and trend-confirmation indicator.

3.2 Cross-Maturity Quantum Entanglement

Near-instantaneous stress transmission from front-VIX microstructure to back-VIX microstructure resembles non-local quantum entanglement more closely than slow, classical arbitrage diffusion. We define the cross-maturity Entanglement Synchronization index $E_{ent}(t)$ as:

$$E_{ent}(t) = \cos\left(\phi_{short}(t) - \phi_{long}(t)\right)$$

where ϕ_{short} and ϕ_{long} represent the instantaneous Hilbert phase angles of short-dated $\phi_{short}(t)$ and long-dated $\phi_{long}(t)$ futures contracts, respectively. An empirical threshold—the Critical Decoherence Line at $E_{ent} = 0.8$—operationalizes systemic contagion risk.

Rather than fixing the split at the first k contracts, we assign each contract to the short- or long-end based on its time-to-maturity (TTM). Contracts with $\mathrm{TTM}_j(t) < \tau$ are averaged to form the short-end wave function $\psi_S(t)$; the remainder form the long-end wave function $\psi_L(t)$. We set τ=30 days, corresponding to the boundary between front-VIX microstructure and back-VIX microstructure. This makes the split time-adaptive, and no information is lost at rollover. The entanglement index is then

$$\mathrm{Sync}(t) = \sin^2 \left(\frac{\phi_S(t)-\phi_L(t)}{2}\right),$$

where ϕ_S, ϕ_L are the unwrapped phases of ψ_S, ψ_L obtained via the Hilbert transform. A value near 1 indicates decoherence between short-term panic and long-term expectations (typically during crashes); near 0 indicates phase-locked markets (calm periods).

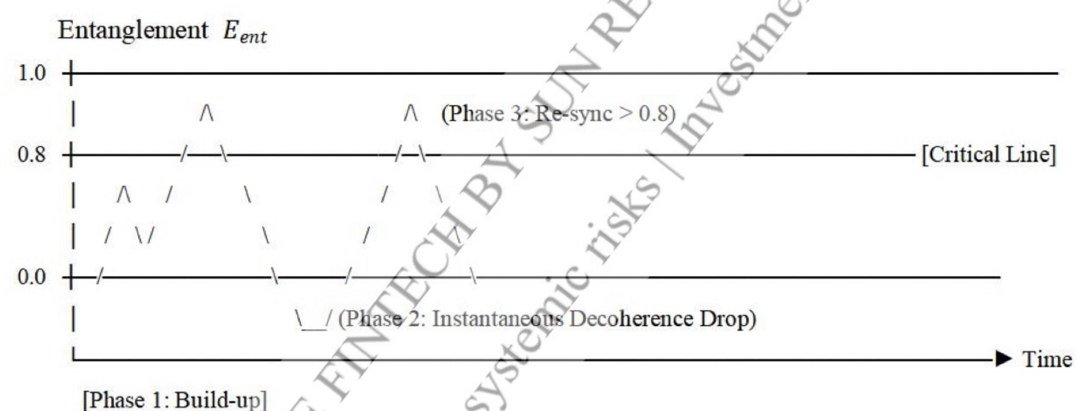

Fig 2. The Three-Phase Entanglement Sequence $E_{ent}(t)$ Schematic Illustration

3.3 The Three-Phase Entanglement Sequence

Around discontinuous market shocks, $E_{ent}(t)$ follows a three-phase dynamic pattern:

➢ Pre-Shock Entanglement Build-Up: In the 1–4 weeks prior to a shock, $E_{ent}(t)$ repeatedly touches and crosses above the 0.8 Critical Decoherence Line, indicating tightening cross-maturity correlation.

➢ Instantaneous Measurement Decoherence: At the exact moment of an unexpected price gap or jump shock, $E_{ent}(t)$ drops sharply. This mirrors quantum measurement theory: the shock acts as an external measurement that breaks the existing phase relationship, collapsing the wave function.

➢ Post-Shock Re-Synchronization: Following the initial shock, $E_{ent}(t)$ rapidly bounces back above 0.8 and remains elevated. This indicates that the microstructure has settled into a new, highly entangled high-stress state.

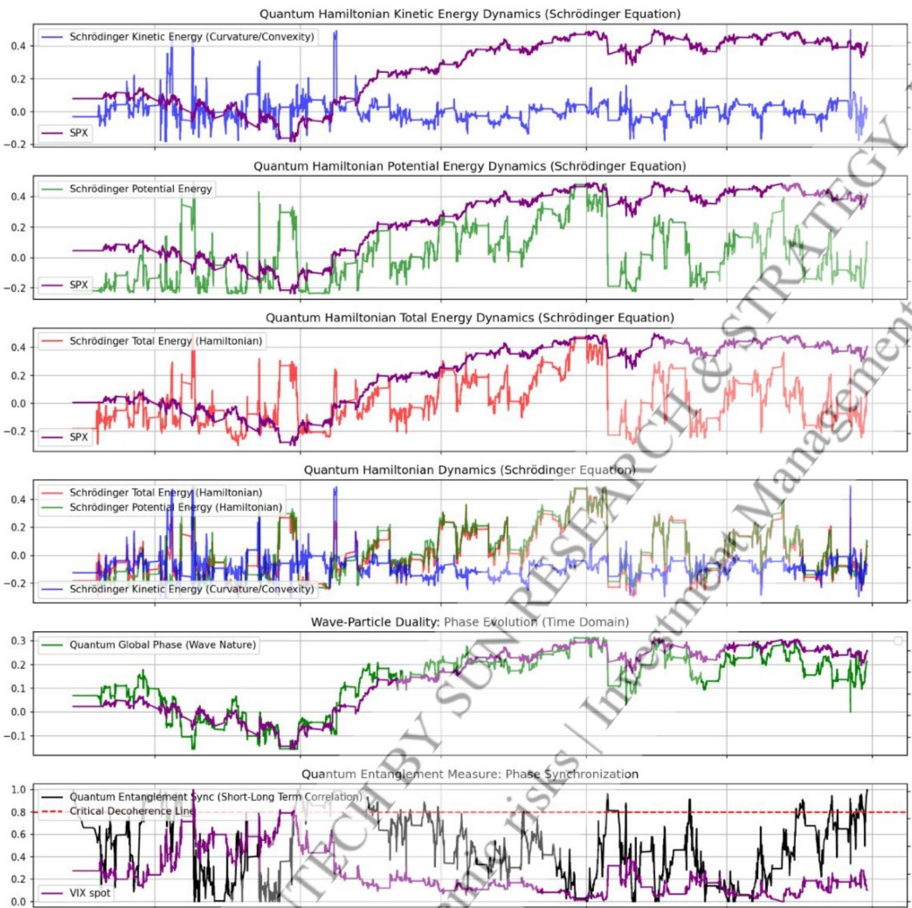

Fig 3. Minute Level Illustrations of The Quantum Mechanical Framework's State Variables From 2026.01-08

Discussion

4. Unified Thermodynamic-Quantum Systemic Risk Architecture

To address false positives inherent in single-variable indicators (such as the January 2024 Gibbs free energy trough), our previous paper and this paper are integrated into a multi-layer confirmation framework.

4.1 The Four-Layer Multi-Indicator Confirmation Rule

Layer	Source Framework	Primary Variable	Systemic Function
1. *Trend*	Paper II (Quantum)	Global Phase Slope $\left(\frac{d\Phi}{dt}\right)$	Directional Trend Confirmation
2. *Stability*	Paper I (Thermodynamic)	Gibbs Free Energy (G)	Overall Stress Level & Kinematics
3. *Contagion*	Paper II (Quantum)	Entanglement Sync $(E_{ent} \geq 0.8)$	Cross-Maturity Stress Transmission
4. *Liquidity*	Paper I/II (Kinetic)	Market Viscosity (η)	Liquidity Freeze / Quote Gelling

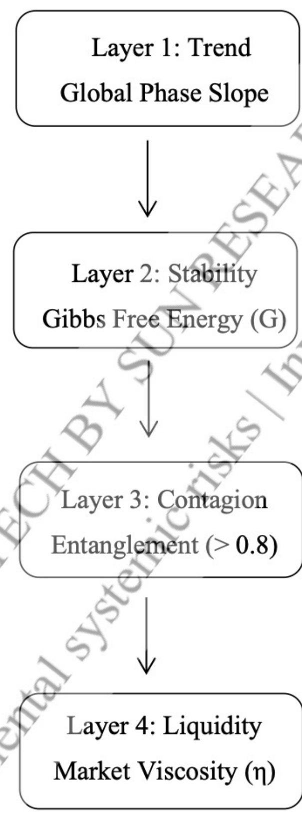

Fig 4.The Four-Layer Multi-Indicator Confirmation Rule Schematic Illustration

The motivating empirical observation is that these four variables were derived independently — G from a classical thermodynamic identity within our previous paper, Phase and Entanglement from a quantum wave-mechanical construction in this paper, Viscosity from a kinetic-theory analogy accompanying both — yet cluster around the same historical episodes (August 2024, April 2025, March 2026) identified independently in our two companion reviews. We regard convergence across independently constructed instruments as a form of informal triangulation, analogous to multiple imperfect sensors agreeing; it is, in our view, a meaningfully stronger basis for confidence than any single one of these variables taken alone, and it directly addresses the false-positive problem we documented for G in isolation in our previous paper (the January 2024 episode with no matching SPX drawdown). Under the proposed rule, a window is flagged high-confidence only when at least two of the stability, contagion, and liquidity layers confirm simultaneously, with the trend layer used separately to calibrate the size rather than the presence of a risk-reduction response.

4.2 Multi-Indicator Triangulation Logic

A high-confidence systemic risk warning requires simultaneous confirmation across at least two of the three risk layers (Stability, Contagion, Liquidity):

➢ Risk Accumulation Phase: T_{eff} oscillation amplitude expands, S(t) decreases, and $E_{ent}(t)$ breaches 0.8, while G(t) remains within its baseline.

➢ High-Confidence Warning Trigger: G(t) breaks out into a deep negative trough or spikes positive, accompanied by Market Viscosity clustering in the 0.8–1.0 range and Entanglement $E_{ent} \geq 0.8$.

➢ Execution Scaling via Trend Layer: Position sizing is scaled using Layer 1 (Global Phase). If $\frac{d\Phi}{dt} > 0$, equity exposure is partially hedged rather than fully liquidated. If $\frac{d\Phi}{dt} < 0$, defensive deleveraging is executed in full.

➢ Re-Entry Confirmation: Rebuilding equity exposure requires G(t) and S(t) to mean-revert toward baseline levels, $E_{ent}(t)$ to drop below 0.8, and Global Phase $\Phi(t)$ to resume an upward trajectory.

5. Toward an Integrated Risk and Timing Application

A practical system[1-5] built on this work would compute: (a) the sign and slope of Global Phase, (b) G's position relative to its historical baseline and whether it is approaching the critical line from below or from a deep trough, (c) whether Entanglement Sync has persistently — not just momentarily — exceeded 0.8, and (d) whether Market Viscosity is clustering at elevated levels while the Boltzmann relaxation rate is simultaneously depressed. A candidate SPY-exposure rule would reduce risk when two or more of (b)–(d) confirm, size the reduction using (a) so that strong uptrends are only partially hedged rather than fully exited, and treat re-entry as confirmed only once Entanglement Sync has receded from above 0.8 and Phase has resumed an upward slope. Our empirical analysis yields several significant results:

➢ Wave-Particle Duality: The phase evolution of the financial wave function captures market sentiment and trend changes, demonstrating wave-like properties.

➢ Quantum Entanglement: High entanglement values between short and long-term VIX futures often precede market stress, indicating synchronized market behavior.

➢ Quantum Tunneling Effects: Rapid transitions in market state, resembling quantum tunneling, are observed during periods of high volatility.

6. What Is Novel in this research paper

A genuine change of mathematical register, not just new variables. Our previous paper is a state-function (equilibrium) description: S, H, T, G are computed from a snapshot of the VIX microstructure. This work is a dynamical description: the wave function evolves under an explicit differential equation, so the model in principle carries information about how the microstructure moves between snapshots, not only what it looks like at each snapshot — a strictly richer object.

Decomposition of microstructure instability into a curvature (kinetic) component and a holding-cost (potential) component. Where previous paper's H folds VIX level and VVIX pressure into one enthalpy term, this work's Hamiltonian separates shape instability (curvature) from carry cost (the potential well), giving two independently interpretable channels for the same underlying stress.

A non-local, cross-maturity contagion measure with an explicit threshold. Our previous paper's state variables are computed within a single microstructure at a single time; they do not directly measure whether stress at one maturity is transmitting to another. This paper's entanglement score is explicitly a between-maturity measure, and the addition of a numerical Critical Decoherence Line (0.8) converts this paper's qualitative claim into an operational, chartable threshold — a concrete advance over the purely descriptive statement in the original text.

7. Empirical findings from the comprehensive historical regime spectrum

Global Phase as a trend-confirmation layer. Across essentially all regime spectrum, Global Phase and the SPX price path move together, sharing local turning points with striking consistency. This is qualitatively different from our previous paper's G, whose value is only loosely, non-monotonically related to price level (G is a stability judgment, not a direction judgment). We propose that Phase is best used as an independent trend-confirmation signal rather than as a risk-warning signal.

Hamiltonian bursts as visual evidence for the tunneling analogy. At the sharpest, most discontinuous shocks in the sample — the August 5, 2024 unwind, the April 2025 tariff shock, and the March 2026 escalation, all previously identified in our previous paper's review — Kinetic Energy and Total Energy move to extreme values essentially simultaneously, in a step-like rather than gradual fashion. In gradual, grinding deteriorations (e.g., the 2023 rate-driven correction), the same panel instead shows smoother, more continuous drift. This distinction between step-like and gradual Hamiltonian behavior lines up with, and offers an independent variable supporting, the Shock Kinematics Hypothesis proposed in our previous paper (gradual shocks producing G troughs; discontinuous shocks producing G spikes).

The three-phase entanglement pattern. Around the clearest discontinuous shocks, Entanglement Sync shows a repeatable sequence: (i) a multi-week build-up phase in which touches and short stays above the 0.8 line become more frequent; (ii) at the instant of the sharpest repricing, a brief, sharp drop in synchronization; and (iii) rapid re-synchronization at a sustained level above 0.8 in the shock's immediate aftermath. We find the physical reading of stage (ii) genuinely apt rather than merely decorative: a discontinuous repricing functions as a "measurement" that momentarily disrupts the phase relationship the model has been tracking, after which the system re-settles into a new, more tightly correlated regime — the market's own analogue of decoherence followed by re-entanglement at a different energy level.

Conclusion

This paper presents a novel quantum mechanical framework for VIX microstructure analysis. The application of quantum concepts provides new insights into market dynamics and offers potential for developing advanced risk management tools. This work's contribution is to recast the VIX microstructure's evolution, not merely its instantaneous state, in the language of quantum mechanics, adding a genuinely dynamical, cross-maturity, non-local layer (entanglement) that our previous paper's equilibrium thermodynamic description does not provide by construction. Combined with our previous work, and validated by the convergence of independently constructed variables around the same historical shock episodes, this supports a four-layer integrated framework for systemic risk assessment that is more robust to any single indicator's false positives than either paper's variables used in isolation. Future research could explore the practical applications of this framework in portfolio and risk assessment.

References:

1. Huzi Sun. Thermodynamic Framework for VIX Microstructure Analysis for Systemic Risk Management: Entropy, Enthalpy, and Gibbs Free Energy. **2026**, PREPRINT.
2. Huzi Sun. Systematic Integration of systemic risk management and Multi-Factor Pricing Dynamics in S&P 500 Price Discovery. *SSRN*, **2026**, http://dx.doi.org/10.2139/ssrn.7223479.
3. Huzi Sun. State of the Art Macro Risk Controlled Risk Parity Model for Macro Multi-Strategy Funds. **2026**, PREPRINT.
4. A global asset allocation system based on a macro alternative multi-factor rating model. *Patent number: CN202511140743.3A*
5. An all-weather global asset allocation system based on macro alternative multi-factor rating models. *Global PCT Patent, Application No. PCT/CN2026/112498*

Competing interests: †Core technical implementation protected by Chinese Patent Application No.CN202511140743.3A (Filed: 2025-08-14) and Patent Cooperation Treaty (PCT) international patent application No. PCT/CN2026/112498 (Filed: 2026-07-22). Full patent documentation available via CNIPA/WIPO.

Data, code, and materials availability: All data, code, and materials used in the analysis are available upon requests and signing the materials transfer agreements (MTAs) for purposes of reproducing or extending the analysis.